SHENTON
COLLEGE

ATMAS Mathematics Specialist

Test 2 Calculator Free

Name: ....

Teacher: Mr Smith

Time Allowed : 30 minutes

Marks	/28

Materials allowed: No special materials.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given in exact values.
Marks may not be awarded for untidy or poorly arranged work.

1 Determine which of the following functions are one-to-one. For those functions which are one-to-one, find their inverse. (6)

a) $(x-2)^2 + (y+3)^2 = 16$

Not one-to-one.

b) $y = \dfrac{1}{x-2}$

$x = \dfrac{1}{y-2}$

$y - 2 = \dfrac{1}{x}$

$y = \dfrac{1}{x} + 2.$

c) $(x-2)^2 + 4$

Not one-to-one.

d) $(x-2)^3 + 4$

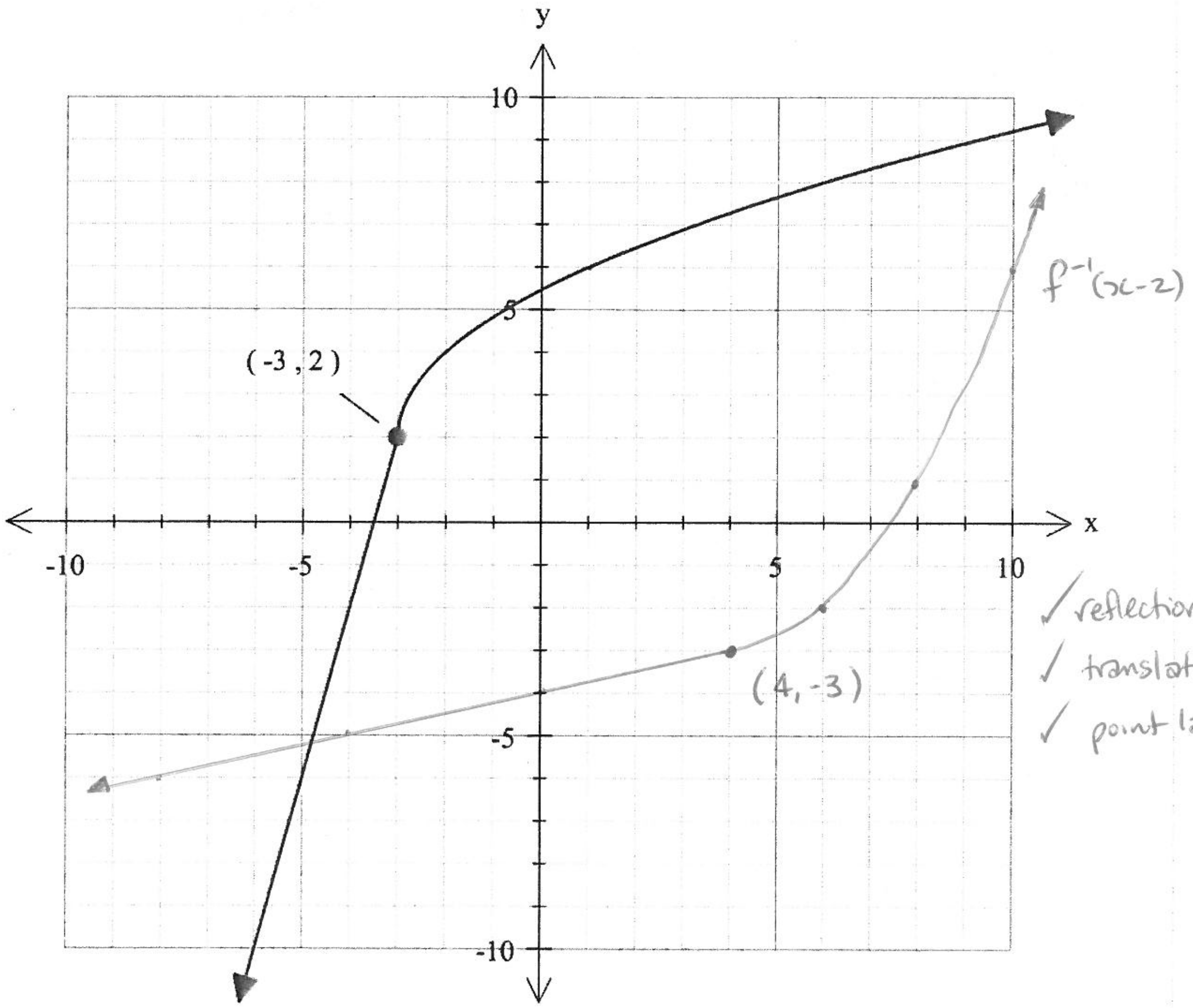
$(y-2)^3 + 4 = x$

$(y-2)^3 = x - 4$

$y - 2 = \sqrt[3]{x-4}$

$y = \sqrt[3]{x-4} + 2.$

2 Below is a graph of $y = f(x)$. On the same set of axes, sketch the graph of $y = f^{-1}(x-2)$. (3)



✓ reflection
✓ translation
✓ point label.

3 If $f(x) = \sin 2x$ and $g(x) = x^2 + 2$,
Determine $g(f(x))$, giving the domain and range of the composition. (3)

$\sin 2x$ D: $x \in \mathbb{R}$.
R: $-1 \leq y \leq 1$

$(\sin 2x)^2 + 2$ D: $-1 \leq x \leq 1$
R: $2 \leq y \leq 3$

✓ D & R of $f(x)$
✓ brigde R → D.
✓ R of composite

4 If $f(x) = e^x$ and $g(x) = \frac{1}{x-1}$,

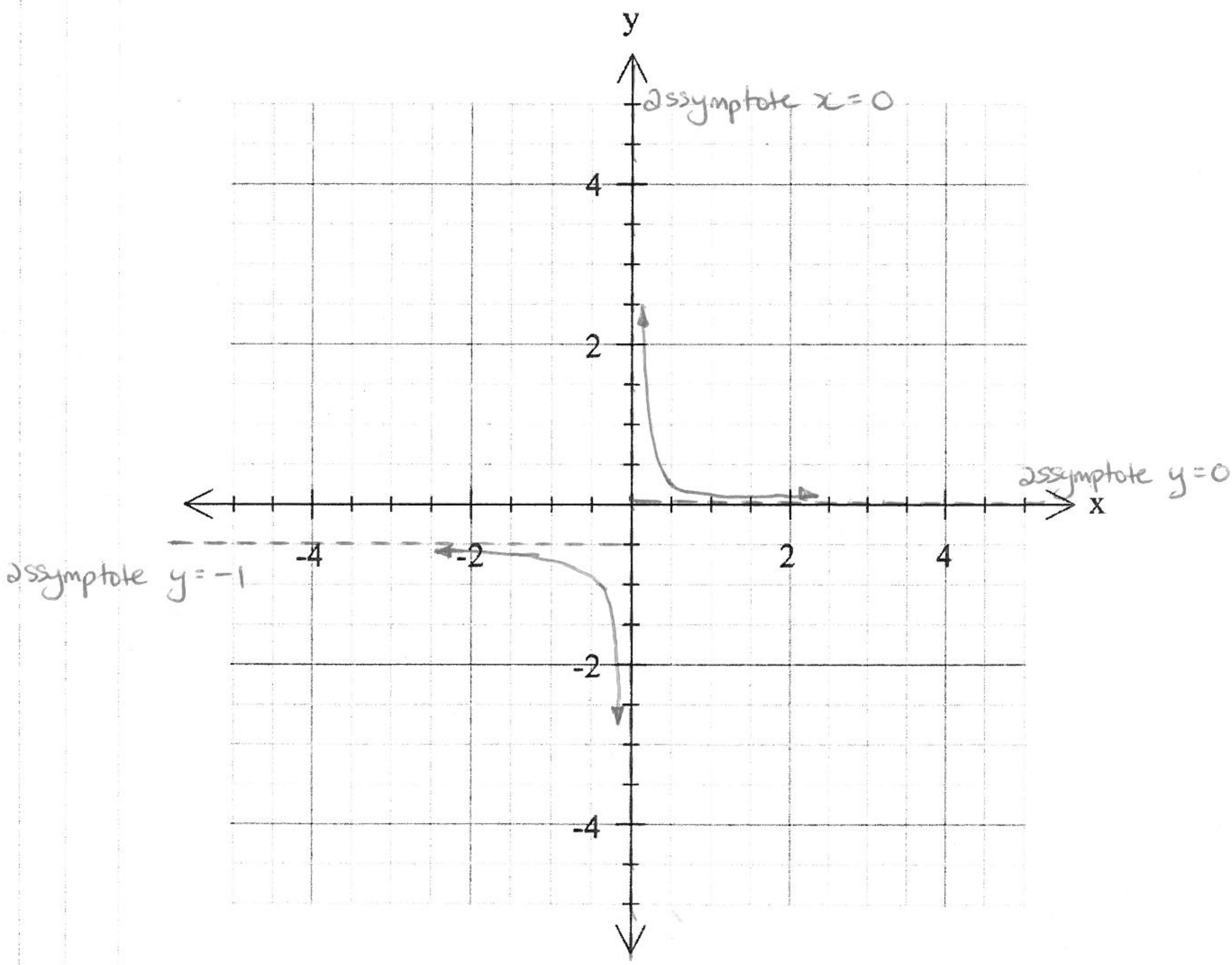
a) Determine $g(f(x))$, giving the domain and range of the composition. (3)

$g \circ f(x) = \frac{1}{e^x - 1}$

D: $x \in \mathbb{R},\ x \neq 0$

R: $y < -1,\ y > 0$

b) Draw a rough sketch of the composite function $y = g(f(x))$, indicating any important features. (3)



5 If $h(x) = \frac{1}{4^x}$ and $h(k(x)) = 2^{2-2x-2x^2}$, find the equation of $k(x)$. (4)

$h(x) = 2^{-2x}$

$\Rightarrow h(k(x)) = 2^{-2k(x)}$

$\Rightarrow -2k(x) = 2 - 2x - 2x^2$

$\Rightarrow k(x) = x^2 + x - 1$

✓✓ relate $h(x)$ to compound

✓ equate powers

✓ solve for $k(x)$

6 The function f(x) is defined as $f(x) = |x + 1| + |x - 2|$.

a) Complete the following… (3)

$$f(x) = \begin{cases} \underline{-2x+1} & \text{for } x < -1 \\ \underline{3} & \text{for } -1 \le x \le 2 \\ \underline{2x-1} & \text{for } x > 2 \end{cases}$$

$x < -1$.

$-(x+1) + -(x-2)$

$= -2x + 1$

$-1 \le x \le 2$.

$(x+1) + -(x-2)$

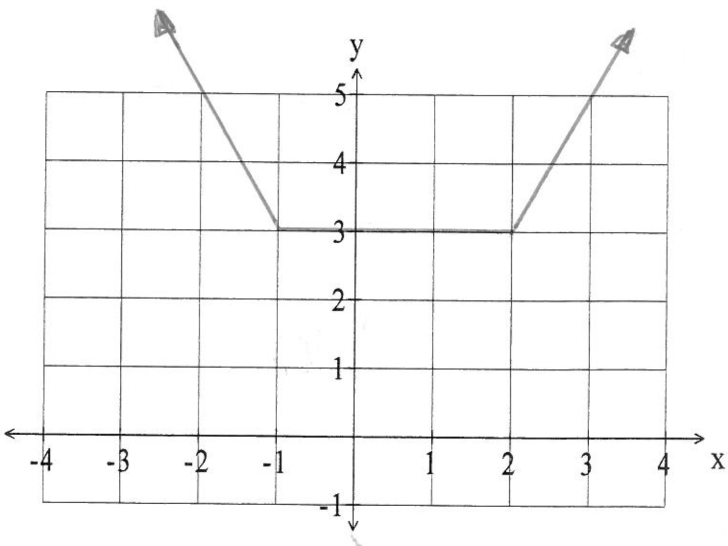
$= 3$

$x > 2$.

$(x+1) + (x-2)$

$= 2x - 1$

b) Sketch the function $f(x) = |x + 1| + |x - 2|$ on the set of axes below. (3)



ATMAS Mathematics Specialist

Test 2 Calculator Assumed

SHENTON COLLEGE

Name: ....

Teacher: Mr Smith

Time Allowed : 25 minutes

Marks	/34

Materials allowed: Classpad, calculator.

Attempt all questions.

All necessary working and reasoning must be shown for full marks.

Where appropriate, answers should be given to two decimal places.

Marks may not be awarded for untidy or poorly arranged work.

1 For a line passing through the point $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and parallel to the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, find

a) The vector equation of the line. (2)

$\tilde{r} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

b) The parametric equations of the line. (2)

$x = 3 - 2\lambda$

$y = -4 + \lambda$

c) The Cartesian equation of the line. (2)

$\lambda = y + 4$

$x = 3 - 2(y+4)$

$x = 3 - 2y - 8$

$x + 2y + 5 = 0$

or $y = \frac{-1}{2}x - \frac{5}{2}$.

2 Line L_1 has the vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Find the equation of L_2, a line perpendicular to L_1 and passing through position $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. (2)

$\tilde{r} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ or similar

3 Draw a sketch of each of the following rational functions, indicating on your sketch important features such as asymptotes, intercepts, and critical points.

- You may use your Classpad to find intercepts, these do not need to be shown algebraically.
- You may also use your Classpad to calculate any derivates required, however, you must then clearly show how you would interpret the relevant calculus to assist you with your sketch.

a) $y = \dfrac{2x^2}{x+2}$ (6)

y-int at $(0,0)$.

vertical assymptote at $x=-2$.

$$y = \frac{2x^2+4x}{x+2} - \frac{4x+8}{x+2} + \frac{8}{x+2}$$

$$= 2x - 4 + \frac{8}{x+2}$$

or synthetic

$$\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ & -4 & 8 & \\ \hline 2 & -4 & 8 & \end{array}$$

$2x - 4 + \frac{8}{x+2}$

=> oblique assymptote of $y = 2x-4$.

$$\frac{dy}{dx} = \frac{2x^2+8x}{(x+2)^2}$$

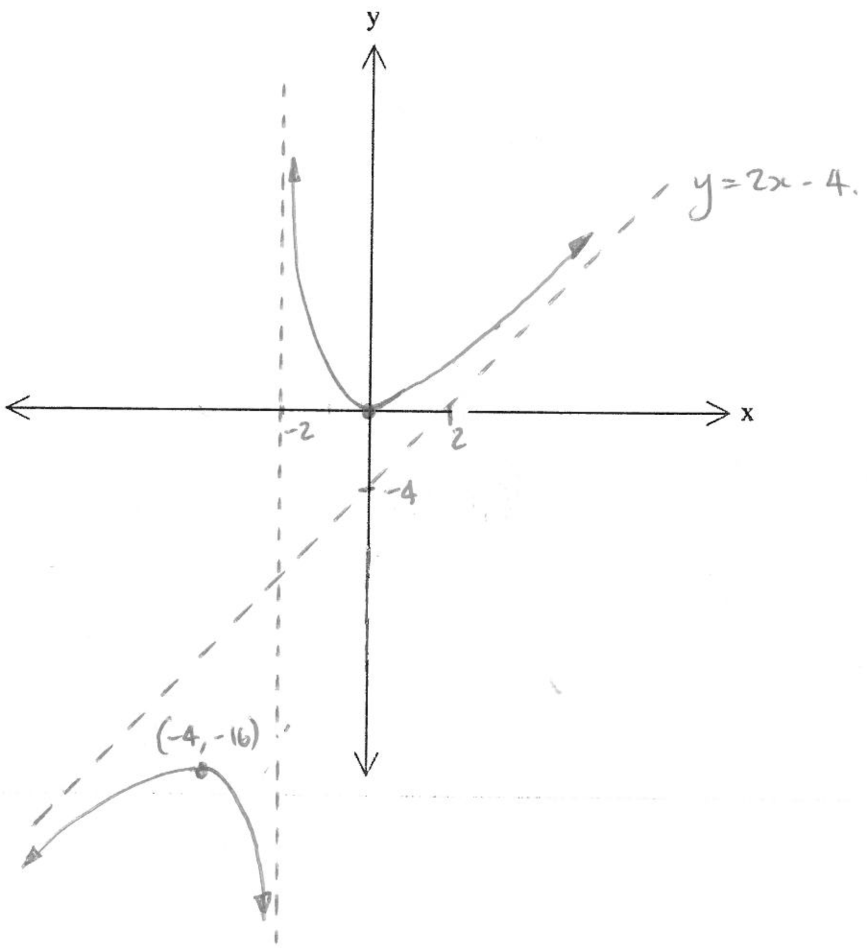
$2x^2 + 8x = 0$

=> $x = 0$ or $x = -4$

(nature & y-coord from CAS)

=> Max at $(-4,-16)$

min at $(0,0)$



✓ vertical found

✓✓ oblique found.

✓ use of derivative = 0

✓ stationary points.

✓ graph.

b) $y = \dfrac{x+1}{x-1}$ (5)

vertical assymptote at $x = 1$.

$$y = \frac{x-1}{x-1} + \frac{2}{x-1}$$

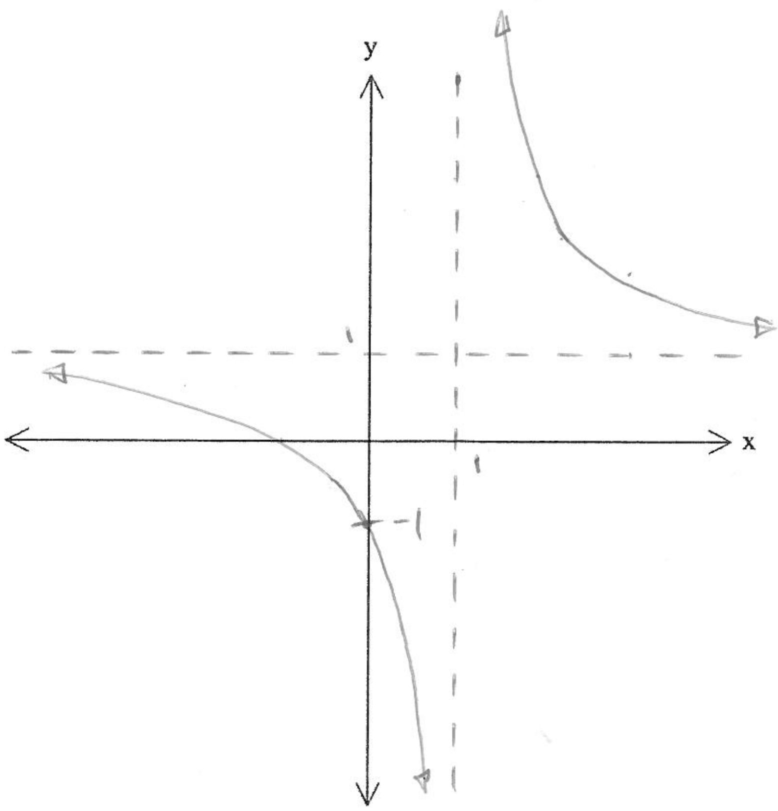
$$= 1 + \frac{2}{x-1}$$

=> horizontal assymptote at $y = 1$.

$$\frac{dy}{dx} = \frac{-2}{(x-1)^2}$$

=> no stationary points

y-int $(0, -1)$.



✓ vertical at 1.

✓ show horizontal

✓ show no stationary

✓✓ graph + point.

c) $y = \dfrac{x+2}{(x-1)(x+4)}$ (6)

vertical assymptotes at $x = 1$ and $x = -4$.

$$\frac{dy}{dx} = \frac{x^2 + 4x + 10}{(x^2 + 3x - 4)^2}$$

$x^2 + 4x + 10 = 0$

no real solutions

$\Rightarrow$ no stationary points

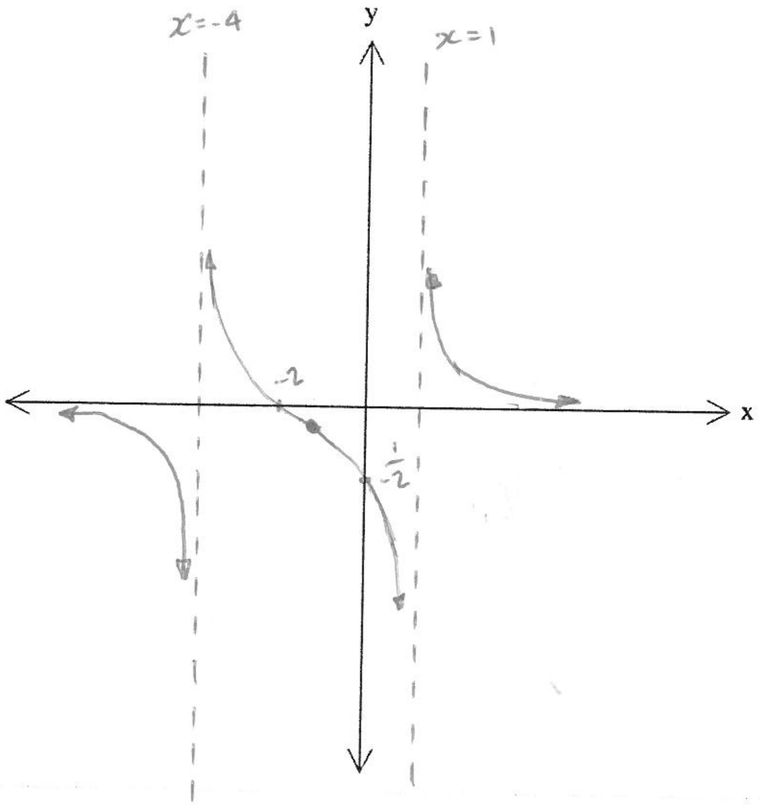
$$\frac{d^2y}{dx^2} = 0$$

$\Rightarrow x \approx -1.67$

vertical inflection at $x = -1.67$

as $x \to \infty$, $y \to 0^+$

as $x \to -\infty$, $y \to 0^-$



✓ vertical

✓ show no stationary.

✓ inflection.

✓ end limits

✓✓ graph + point

5 Three snooker players, Eddie, Neil and Warren, are trying to set up a trick shot for a competition. The black ball is placed at certain position on the table. When a subtle signal is given, all three players shoot at exactly the same time and aim their shots to hit the black exactly 0.6 seconds later.

Neil starts from position $\begin{pmatrix} -30 \\ -14 \end{pmatrix}$ and shoots with velocity $\begin{pmatrix} 60 \\ 40 \end{pmatrix}$.

Warren intends to use a velocity of $\begin{pmatrix} -90 \\ -30 \end{pmatrix}$, but has yet to determine the correct starting position.

The third player, Eddie, plans to start his shot at $\begin{pmatrix} -45 \\ 49 \end{pmatrix}$ but has yet to refine the correct velocity with which he should shoot.

[As always, assume the balls are points with zero radius for the purposes of your calculations.]

(9)

a) Determine an appropriate vector for Warren's position and Eddie's velocity so that the three players can complete the trick successfully. (7)

Point of collision $\begin{pmatrix} -30 \\ -14 \end{pmatrix} + 0.6\begin{pmatrix} 60 \\ 40 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 10 \end{pmatrix}$. ✓✓ Use Neil to find collision point.

Warren. $\begin{pmatrix} x \\ y \end{pmatrix} + 0.6\begin{pmatrix} -90 \\ -30 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$ ✓ equation

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 60 \\ 28 \end{pmatrix}$ Warren's position is $\begin{pmatrix} 60 \\ 28 \end{pmatrix}$. ✓ solve

Eddie $\begin{pmatrix} -45 \\ 49 \end{pmatrix} + 0.6\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$ ✓ equation

$\begin{pmatrix} p \\ q \end{pmatrix} = \frac{10}{6}\begin{pmatrix} 51 \\ -39 \end{pmatrix}$ ✓ solve

$= \begin{pmatrix} 85 \\ -65 \end{pmatrix}$ Eddie's velocity is $\begin{pmatrix} 85 \\ -65 \end{pmatrix}$.

✓ answers.

b) Determine the angle between Neil and Warren's shots when they hit the black ball. (2)

Using direction vectors $\tilde{n} = \begin{pmatrix} 60 \\ 40 \end{pmatrix}$ and $\tilde{w} = \begin{pmatrix} -90 \\ -30 \end{pmatrix}$

$\tilde{n} \cdot \tilde{w} = |\tilde{n}||\tilde{w}|\cos\theta$. ✓ use of dot product

$-6600 = (20\sqrt{13})(30\sqrt{10})\cos\theta$ ✓ angle.

$\theta = 164.74$

Angle $\simeq 165°$